Implications of Leptonic Unitarity Violation at Neutrino Telescopes

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Abstract

A measurement of the ultrahigh-energy (UHE) cosmic neutrinos at a km^3-size neutrino telescope will open a new window to constrain the 3×3 neutrino mixing matrix V and probe possible new physics. We point out that it is in principle possible to examine the non-unitarity of V, which is naturally expected in a class of seesaw models with one or more TeV-scale Majorana neutrinos, by using neutrino telescopes. Considering the UHE neutrinos produced from the decays of charged pions arising from pp and (or) $p\gamma$ collisions at a distant astrophysical source, we show that their flavor ratios at a terrestrial neutrino telescope may deviate from the democratic flavor distribution $\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = 1 : 1 : 1$ due to the seesaw-induced unitarity violation of V. Its effect can be as large as several percent and can serve for an illustration of how sensitive a neutrino telescope should be to this kind of new physics.

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1 Introduction

The solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed. In the basis where the flavor eigenstates of charged leptons coincide with their mass eigenstates, the phenomenon of neutrino mixing can simply be described by a 3×3 unitary matrix V which links the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ to the neutrino mass eigenstates (ν_1, ν_2, ν_3):

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{1}
$$

A full parametrization of V requires 3 rotation angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and 3 phase angles (δ, ρ, σ) [5]:

$$
V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & +c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_{\mathrm{M}} , \tag{2}
$$

where $s_{ij} \equiv \sin\theta_{ij}, c_{ij} \equiv \cos\theta_{ij}$ (for $ij = 12, 13, 23$), and $P_{\mathrm{M}} = \mathrm{Diag}\{1, e^{i\rho}, e^{i\sigma}\}$ is the Majorana phase matrix irrelevant to neutrino oscillations. A global analysis of current experimental data [6] points to $\theta_{13} \approx 0$ and $\theta_{23} \approx \pi/4$, a noteworthy result which has motivated a number of authors to consider the μ-τ permutation symmetry and its breaking mechanism for model building [7].

Now that neutrinos can oscillate from one flavor to another, it will be extremely interesting to detect the oscillatory phenomena of ultrahigh-energy (UHE) cosmic neutrinos produced from distant astrophysical sources. IceCube [8], a km^3-volume under-ice neutrino telescope, is now under construction at the South Pole and aims to observe the UHE neutrino oscillations. Together with the under-water neutrino telescopes in the Mediterranean Sea (ANTARES [9], NESTOR [10] and NEMO [11]), IceCube has the potential to shed light on the acceleration mechanism of UHE cosmic rays and to probe the intrinsic properties of cosmic neutrinos. An immediate consequence of neutrino oscillations is that the flavor composition of cosmic neutrinos to be observed at the telescopes must be different from that at the sources [12]. By measuring the cosmic neutrino flavor distribution, one can determine or constrain the mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and the Dirac CP-violating phase (δ). A lot of attention has recently been paid to this intriguing possibility [13]—[16].

We aim to investigate the oscillation of cosmic neutrinos produced from the decays of charged pions arising from energetic pp and (or) $p\gamma$ collisions at a distant astrophysical source (e.g., active galactic nuclei or AGN). For such a most probable UHE neutrino source, its flavor composition is

$$
\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0 , \tag{3}
$$

where $\phi_\alpha \equiv \phi_{\nu_\alpha} + \phi_{\bar{\nu}_\alpha}$ (for $\alpha = e, \mu, \tau$) denotes the α-neutrino flux at the source. As the distances between the astrophysical sources and the terrestrial detectors are much longer than the typical length of solar or atmospheric neutrino oscillations, one may average the UHE cosmic neutrino oscillation probabilities and arrive at

$$
P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2 . \tag{4}
$$

This result is also valid for the anti-neutrino oscillations; namely, $\bar{P}_{\alpha\beta} \equiv P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P_{\alpha\beta}$ for $\alpha, \beta = e, \mu$ and τ. Therefore, the neutrino fluxes at the detector can be calculated from

$$\phi_\alpha^{\mathrm{T}} = \sum_\beta P_{\alpha\beta} \phi_\beta \ . \tag{5}$$

Given Eq. (3) together with the condition $|V_{\mu i}| = |V_{\tau i}|$ (for $i = 1, 2, 3$) [17], it is easy to show that the flavor distribution of UHE cosmic neutrinos has a democratic pattern at neutrino telescopes:

$$\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = 1 : 1 : 1 \ . \tag{6}$$

Note that $|V_{\mu i}| = |V_{\tau i}|$ implies either $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (CP invariance) or $\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$ (CP violation) in the standard parametrization of V as shown in Eq. (2). These two sets of interesting conditions can be realized from the so-called tri-bimaximal [18] and tetra-maximal [19] neutrino mixing scenarios, respectively.

One has to bear in mind that $\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = 1 : 1 : 1$ depends on two idealized hypotheses: the astrophysical source of UHE neutrinos satisfies $\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0$ and the 3×3 neutrino mixing matrix V satisfies $|V_{\mu i}| = |V_{\tau i}|$. Previous works have extensively analyzed possible deviations from the democratic flavor distribution of UHE cosmic neutrinos at neutrino telescopes by taking account of the energy dependence, uncertainties in the neutrino mixing angles, contaminations to the canonical production of ν_e's ($\bar{\nu}_e$'s) and ν_μ's ($\bar{\nu}_\mu$'s) from $\pi^\pm$'s, and different sources of UHE cosmic neutrinos [13]—[16].

We shall concentrate on the standard pion-decay source of UHE neutrinos, whose flavor composition has been given in Eq. (3), to explore the effects of non-unitarity of V on the flavor distribution of such cosmic neutrinos at a terrestrial neutrino telescope. This investigation is new and makes sense, because V is naturally expected to be non-unitary in a class of seesaw models with one or more TeV-scale right-handed Majorana neutrinos. We find that the democratic flavor distribution in Eq. (6) can be broken at the percent level as a consequence of the unitarity violation of V. Although such a small effect is hard to be observed in any realistic experiments in the foreseeable future, it *does* illustrate how sensitive a neutrino telescope should be to this kind of new physics.

2   Unitarity Violation at Neutrino Telescopes

If the tiny masses of three known neutrinos (ν_1, ν_2, ν_3) are attributed to the popular seesaw mechanism (either type-I [20] or type-II [21]), in which there exist a few heavy (right-handed) Majorana neutrinos N_i, then the 3×3 neutrino mixing matrix V must be non-unitary. The effect of unitarity violation of V depends on the mass scale of N_i, and it can be of $\mathcal{O}(10^{-2})$ if N_i are at the TeV scale [22] — an energy frontier to be explored by the LHC. Indeed, a global analysis of current neutrino oscillation data and precision electroweak data yields some stringent constraints on the non-unitarity of V, but its effect is allowed to be of $\mathcal{O}(10^{-2})$ [23] and may have some novel implications on neutrino oscillations [24]—[26].

In the presence of small unitarity violation, we write the neutrino mixing matrix as $V = AV_0$, where V_0 is a unitary matrix containing 3 rotation angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and 3 phase angles like that

given in Eq. (2), and A is a quasi-identity matrix which can in general be parametrized in terms of 9 rotation angles θ_{ij} and 9 phase angles δ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$) [24]. For simplicity, here we adopt the expression of A shown in Eq. (11) of Ref. [24] and take V_0 to be the well-known tri-bimaximal mixing pattern [18] without any CP-violating phases. Then we obtain the non-unitary neutrino mixing matrix $V = AV_0$ as follows:

$$V = \begin{pmatrix} \frac{2}{\sqrt{6}}(1 - W_1) & \frac{1}{\sqrt{3}}(1 - W_1) & 0 \\ -\frac{1}{\sqrt{6}}(1 - W_2 + 2X) & \frac{1}{\sqrt{3}}(1 - W_2 - X) & \frac{1}{\sqrt{2}}(1 - W_2) \\ \frac{1}{\sqrt{6}}(1 - W_3 - 2Y + Z) & -\frac{1}{\sqrt{3}}(1 - W_3 + Y + Z) & \frac{1}{\sqrt{2}}(1 - W_3 - Z) \end{pmatrix} , \qquad (7)$$

where

$$W_i = \frac{1}{2}\left(s_{i4}^2 + s_{i5}^2 + s_{i6}^2\right) , \qquad (8)$$

for $i = 1, 2, 3$; and

$$\begin{aligned} X &= \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^* , \\ Y &= \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^* , \\ Z &= \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^* . \end{aligned} \qquad (9)$$

Here $s_{ij} \equiv \sin\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}}s_{ij}$ have been defined, and higher-order terms of s_{ij} have been neglected. The mixing angles in θ_{ij} can at most be of $\mathcal{O}(0.1)$, but the CP-violating phases δ_{ij} are entirely unrestricted. If both θ_{ij} and δ_{ij} are switched off, the tri-bimaximal neutrino mixing pattern will be reproduced from Eq. (7). With the help of Eqs. (4) and (7), we arrive at

$$\begin{aligned} P_{ee} &= \frac{5}{9} - \frac{20}{9}W_1 , \\ P_{e\mu} &= \frac{2}{9} - \frac{4}{9}(W_1 + W_2) + \frac{2}{9}\mathrm{Re}X , \\ P_{e\tau} &= \frac{2}{9} - \frac{4}{9}(W_1 + W_3) - \frac{2}{9}(\mathrm{Re}Y - 2\mathrm{Re}Z) , \\ P_{\mu\mu} &= \frac{7}{18} - \frac{14}{9}W_2 - \frac{2}{9}\mathrm{Re}X , \\ P_{\mu\tau} &= \frac{7}{18} - \frac{7}{9}(W_2 + W_3) - \frac{1}{9}(\mathrm{Re}X - \mathrm{Re}Y + 2\mathrm{Re}Z) , \\ P_{\tau\tau} &= \frac{7}{18} - \frac{14}{9}W_3 + \frac{2}{9}(\mathrm{Re}Y - 2\mathrm{Re}Z) . \end{aligned} \qquad (10)$$

For the canonical astrophysical source of UHE neutrinos under consideration, we definitely have $\{\phi_e, \phi_\mu, \phi_\tau\} = \{1/3, 2/3, 0\}\phi_0$, where ϕ_0 denotes the total initial flux. It is then easy to get the flavor distribution at a terrestrial neutrino telescope:

$$\begin{aligned} \phi_e^{\mathrm{T}} &\equiv \frac{\phi_0}{3}\left[1 - \frac{4}{9}(7W_1 + 2W_2) + \frac{4}{9}\mathrm{Re}X\right] , \\ \phi_\mu^{\mathrm{T}} &\equiv \frac{\phi_0}{3}\left[1 - \frac{4}{9}(W_1 + 8W_2) - \frac{2}{9}\mathrm{Re}X\right] , \\ \phi_\tau^{\mathrm{T}} &\equiv \frac{\phi_0}{3}\left[1 - \frac{2}{9}(2W_1 + 7W_2 + 9W_3) - \frac{2}{9}\mathrm{Re}X\right] . \end{aligned} \qquad (11)$$

The democratic flavor distribution of ϕ_α^{T} (for $\alpha = e, \mu, \tau$) is clearly broken. Because of the non-unitarity of V, the total flux of UHE cosmic neutrinos at the telescope is not equal to that at the source:

$$\sum_\alpha \phi_\alpha^{\mathrm{T}} = \phi_0 \left[1 - \frac{2}{3} \left(2W_1 + 3W_2 + W_3 \right) \right] . \tag{12}$$

This sum is apparently smaller than ϕ_0, and it approximately amounts to $0.96\phi_0$ if $W_i \sim 0.01$ (for $i = 1, 2, 3$). Some comments are in order.

(1) Note that $\mathrm{Re}X$ receives the most stringent constraint from current experimental data, $|X| < 7.0 \times 10^{-5}$ [23]. Hence the dominant effects of unitarity violation on ϕ_α^{T} come from W_i. The breaking of $\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = 1 : 1 : 1$ can be as large as several percent. Although the strength of unitarity violation is very small and certainly difficult to be observed in realistic experiments, it *does* illustrate how sensitive a neutrino telescope should be to this kind of new physics.

(2) Note also that the oscillation probabilities of UHE cosmic neutrinos are actually given by $\hat{P}_{\alpha\beta} \equiv P_{\alpha\beta} / \left[(VV^\dagger)_{\alpha\alpha} (VV^\dagger)_{\beta\beta} \right]$ (for $\alpha, \beta = e, \mu, \tau$) in the non-unitary case, where the production of ν_α and the detection of ν_β are both governed by the charged-current interactions [23]. Given the canonical source of UHE neutrinos, ν_e's are generated from the decay of muons, and thus the charged-current interaction involves two lepton flavors (i.e., e and μ). But ν_μ's can be produced from two channels: one is the decay of charged pions and the other is the decay of muons. The former involves only one lepton flavor (i.e., μ). Hence one should take care of the normalization factors when doing specific calculations of the cosmic neutrino fluxes for a specific neutrino-telescope experiment. For the simple pattern of V taken above, the normalization factors can be explicitly written as

$$VV^\dagger = \mathbf{1} - \begin{pmatrix} 2W_1 & X^* & Y^* \\ X & 2W_2 & Z^* \\ Y & Z & 2W_3 \end{pmatrix} . \tag{13}$$

(3) The unitarity violation of V under discussion is ascribed to the existence of heavy Majorana neutrinos in seesaw models and usually referred to as the minimal unitarity violation [23]. In contrast, the existence of one or more light sterile neutrinos and their mixing with three active neutrinos may also violate the unitarity of V. Using $S_{\alpha j}$ to denote the matrix elements of active-sterile neutrino mixing, we can express the averaged probabilities of UHE cosmic neutrino oscillations as

$$P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2 + \sum_{j=1}^{n} |S_{\alpha j}|^2 |S_{\beta j}|^2 , \tag{14}$$

where α and β run over e, μ and τ, and

$$\sum_{i=1}^{3} |V_{\alpha i}|^2 + \sum_{j=1}^{n} |S_{\alpha j}|^2 = 1 , \quad (\text{for } n = 1, 2, \cdots) \tag{15}$$

holds. Eq. (15) shows the apparent unitarity violation of V induced by light sterile neutrinos. Two observations have been achieved in Ref. [27]: (a) for small active-sterile mixing (i.e., $|S_{\alpha j}| \ll 1$), the effect of non-unitarity of V at neutrino telescopes is very small and quite similar to that obtained in

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Eq. (10); (b) for large hitherto-unconstrained mixing between active and sterile neutrino species (i.e., $|S_{\alpha j}| \le 1$), the existence of light sterile neutrinos might significantly modify the democratic flavor distribution of UHE cosmic neutrinos at neutrino telescopes. At present, however, we have to admit that there is no strong experimental or theoretical motivation to introduce light sterile neutrinos into the standard model.

For illustration, we simply assume that there is only one heavy Majorana neutrino, which can be accommodated in the minimal type-II seesaw model [28]. In this case, we are left with three mixing angles $(\theta_{14}, \theta_{24}, \theta_{34})$ and three CP-violating phases $(\delta_{14}, \delta_{24}, \delta_{34})$ characterizing the unitarity violation of V. As done in Ref. [14], three working observables at a neutrino telescope can be defined:

$$
\begin{aligned}
R_e &\equiv \frac{\phi_e^{\mathrm{T}}}{\phi_\mu^{\mathrm{T}} + \phi_\tau^{\mathrm{T}}} \,, \\
R_\mu &\equiv \frac{\phi_\mu^{\mathrm{T}}}{\phi_e^{\mathrm{T}} + \phi_\tau^{\mathrm{T}}} \,, \\
R_\tau &\equiv \frac{\phi_\tau^{\mathrm{T}}}{\phi_e^{\mathrm{T}} + \phi_\mu^{\mathrm{T}}} \,.
\end{aligned}
\tag{16}
$$

In the unitarity limit where V takes the tri-bimaximal mixing pattern, one can easily obtain $R_e = R_\mu = R_\tau = 1/2$, a result which is equivalent to the democratic flavor distribution. With the help of Eqs. (8), (9) and (11), we are able to evaluate the above flux ratios in the presence of unitarity violation:

$$
\begin{aligned}
R_e &\approx \frac{1}{2} - \frac{1}{36} \left[24 s_{14}^2 - 15 s_{24}^2 - 9 s_{34}^2 - 12 s_{14} s_{24} \cos\varrho \right] \,, \\
R_\mu &\approx \frac{1}{2} + \frac{1}{36} \left[12 s_{14}^2 - 21 s_{24}^2 + 9 s_{34}^2 - 6 s_{14} s_{24} \cos\varrho \right] \,, \\
R_\tau &\approx \frac{1}{2} + \frac{1}{36} \left[12 s_{14}^2 + 6 s_{24}^2 - 18 s_{34}^2 - 6 s_{14} s_{24} \cos\varrho \right] \,,
\end{aligned}
\tag{17}
$$

where $\varrho \equiv \delta_{14} - \delta_{24}$ and the higher-order terms of s_{ij} (for $ij = 14, 24, 34$) have been neglected. Taking into account the experimental constraints [23], we have numerically calculated the allowed regions of these working observables in Figure 1, where the phase angle ϱ varies freely in the range $\varrho \in [0, 2\pi]$. Two comments are in order:

- The deviation of R_α (for $\alpha = e, \mu, \tau$) from its value in the unitarity limit (i.e., $R_\alpha = 1/2$) is at most at the 0.1% level. There are two obvious reasons for this result: (a) there exist significant cancellations among the contributions of three mixing angles to the flavor ratios; (b) the mixing angles s_{14} and s_{24} are strictly constrained by $|X| = s_{14} s_{24} < 7.0 \times 10^{-5}$.

- In more general cases with two or three heavy Majorana neutrinos, the above constraint can be loosened. Taking two TeV-scale Majorana neutrinos for example, we can obtain $s_{ij} \sim 0.1$ (for $i = 1, 2, 3$ and $j = 4, 5$) when the destructive interference between $\hat{s}_{14} \hat{s}_{24}^*$ and $\hat{s}_{15} \hat{s}_{25}^*$ terms takes place in X (see Eq. (9) and switch off the contribution of $\hat{s}_{16} \hat{s}_{26}^*$ to X).

While a neutrino telescope is expected to identify different flavors of UHE cosmic neutrinos, it is also expected to measure the total flux as precisely as possible. A notable feature of unitarity violation

of V is that the total flux at the detector is not equal to that at the source, and such a discrepancy may be as large as several percent shown in Eq. (12).

3   Comments on cosmic neutrino decays

So far we have assumed cosmic neutrinos to be stable particles and studied their flavor distribution at neutrino telescopes. Now let us make some comments on cosmic neutrino decays and their possible signatures at neutrino telescopes. It is actually *not* unnatural to speculate that massive neutrinos are unstable and can decay into lighter neutrinos and other massless particles. If neutrino masses arise from spontaneous breaking of the global $(B - L)$ symmetry, for example, then $\nu_j \to \nu_i + \chi$ decays may take place, where χ is a Goldstone particle (i.e., Majoron) [29]. A more exotic scenario, in which massive neutrinos may decay into unparticles, has also been proposed [30].

Here we consider a rather simple case: the decay products of UHE cosmic neutrinos are invisible, implying that the initial neutrinos simply disappear. When the neutrino source spectrum falls with energy in a sufficiently deep way, the daughter neutrino will also have negligible contributions to the total neutrino flux. Then the resultant neutrino flavor distribution at neutrino telescopes is simply given by [16, 31]

$$\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = |V_{e1}|^2 : |V_{\mu 1}|^2 : |V_{\tau 1}|^2 \; , \tag{18}$$

provided ν_1 is the lightest neutrino mass eigenstate (and thus stable). Note that Eq. (18) holds in the assumption that the heavier neutrinos ν_2 and ν_3 completely decay into ν_1 and invisible (massless) particles. If the neutrino mixing matrix V is not unitary, as illustrated in Eq. (7), then the flavor distribution at neutrino telescopes reads

$$\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = 4\,(1 - 2W_1) : (1 - 2W_2 + 4\mathrm{Re}X) : (1 - 2W_3 - 4\mathrm{Re}Y + 2\mathrm{Re}Z) \; . \tag{19}$$

It is straightforward to compute the flavor ratios defined in Eq. (16). In the unitarity limit, we have $R_e = 2$ and $R_\mu = R_\tau = 1/5$; and in the non-unitary case with only one heavy Majorana neutrino, we obtain

$$
\begin{aligned}
R_e &\approx 2 - \left[2s_{14}^2 - s_{24}^2 - s_{34}^2 + 4s_{14}s_{24}\cos\varrho - 4s_{14}s_{34}\cos\vartheta + 2s_{24}s_{34}\cos(\varrho - \vartheta)\right] \; , \\
R_\mu &\approx \frac{1}{5} + \frac{1}{25}\left[4s_{14}^2 - 5s_{24}^2 + s_{34}^2 + 20s_{14}s_{24}\cos\varrho + 4s_{14}s_{34}\cos\vartheta - 2s_{24}s_{34}\cos(\varrho - \vartheta)\right] \; , \\
R_\tau &\approx \frac{1}{5} + \frac{1}{25}\left[4s_{14}^2 + s_{24}^2 - 5s_{34}^2 - 4s_{14}s_{24}\cos\varrho - 20s_{14}s_{34}\cos\vartheta + 10s_{24}s_{34}\cos(\varrho - \vartheta)\right] \; , \quad (20)
\end{aligned}
$$

where $\varrho \equiv \delta_{14} - \delta_{24}$, $\vartheta \equiv \delta_{14} - \delta_{34}$, and higher-order terms of s_{ij} have been neglected. The allowed regions of three flavor ratios are plotted in Figure 2, where the phase angles ϱ and ϑ vary freely in the range $[0, 2\pi]$. Two comments are in order:

- Different from the case discussed in section 2, here the deviation of R_e from its value in the unitarity limit (i.e., $R_e = 2$) can be as large as 4%. In comparison, the deviation of R_μ or R_τ from its value in the unitarity limit (i.e., $R_\mu = R_\tau = 0.2$) can be at the 0.1% level.

- It is worth mentioning that additional terms involving $\mathrm{Re}Y$ and $\mathrm{Re}Z$ are present in Eq. (19), compared to Eq. (11). On the other hand, since s_{14} or s_{24} is confined to a very small value, the non-unitary CP-violating phase ϱ can hardly affect the flavor ratios in Eq. (17). In the decay scenario, however, both the phases ϱ and ϑ can significantly contribute to R_α.

We see that the flavor distribution of UHE cosmic neutrinos in the decay scenario is quite different from that in the standard neutrino oscillation picture. In particular, the democratic flavor distribution of UHE cosmic neutrinos at neutrino telescopes is badly broken even if the condition $|V_{\mu i}| = |V_{\tau i}|$ (for $i = 1, 2, 3$) is satisfied.

4   Summary

Assuming that UHE cosmic neutrinos are produced from the decays of charged pions arising from energetic pp and (or) $p\gamma$ collisions at a distant astrophysical source, one may expect a democratic flavor distribution $\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}} = 1 : 1 : 1$ at neutrino telescopes if either $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (CP invariance) or $\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$ (CP violation) are satisfied in the standard parametrization of V. A lot of attention has been focused on small perturbations to the above conditions such that the resultant flavor distribution is no more democratic. We have explored a novel possibility, in which V is non-unitary and its non-unitarity is induced by heavy Majorana neutrinos as expected in a class of TeV-scale seesaw models, to examine the flavor distribution of UHE cosmic neutrinos at a terrestrial neutrino telescope. We have shown that the effect of unitarity violation on the flavor ratios $\phi_e^{\mathrm{T}} : \phi_\mu^{\mathrm{T}} : \phi_\tau^{\mathrm{T}}$ can be as large as several percent. We have also made some brief comments on cosmic neutrino decays and illustrated the relevant flavor distributions at neutrino telescopes.

A measurement of the flavor distribution of UHE cosmic neutrinos is certainly a big challenge to IceCube and other neutrino telescopes. In the long run, however, we hope that neutrino telescopes can play an interesting role complementary to the terrestrial neutrino oscillation experiments in understanding the intrinsic properties of massive neutrinos and probing possible new physics.

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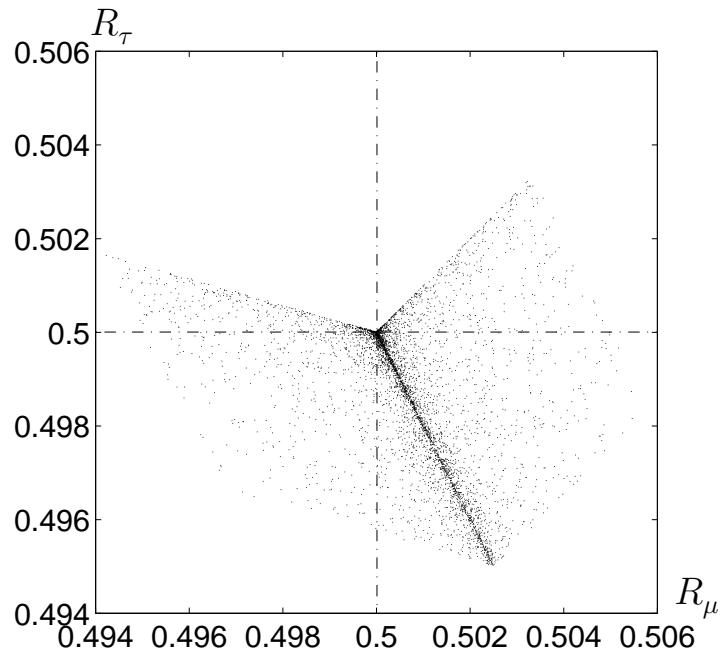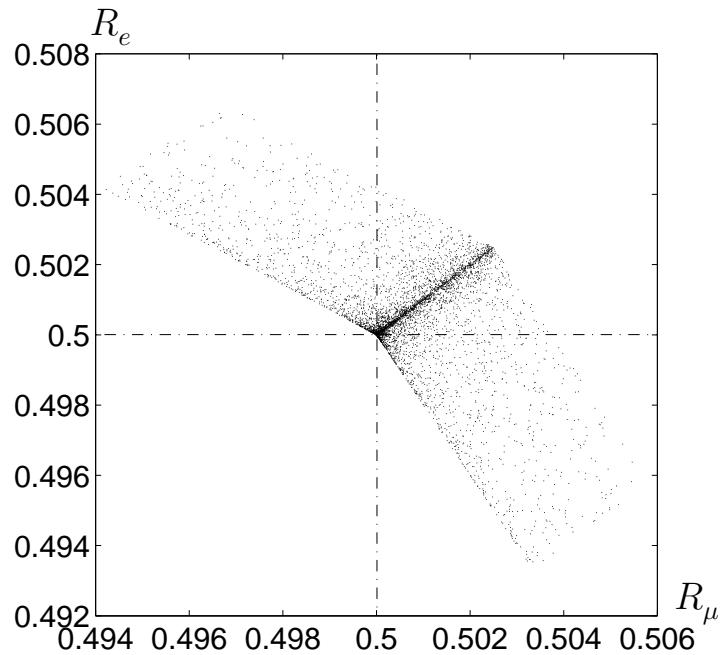
Figure 1: Allowed regions of the flavor ratios $(R_e,\ R_\mu)$ and $(R_\tau,\ R_\mu)$, where the density of points is generated by scanning the possible ranges of s_{ij} (for $ij = 14, 24, 34$) according to a flat random number distribution (i.e., $s_{ij} \in [0, 0.1]$ and $s_{14}s_{24} < 7.0 \times 10^{-5}$ based on current experimental constraints on the non-unitarity of V).
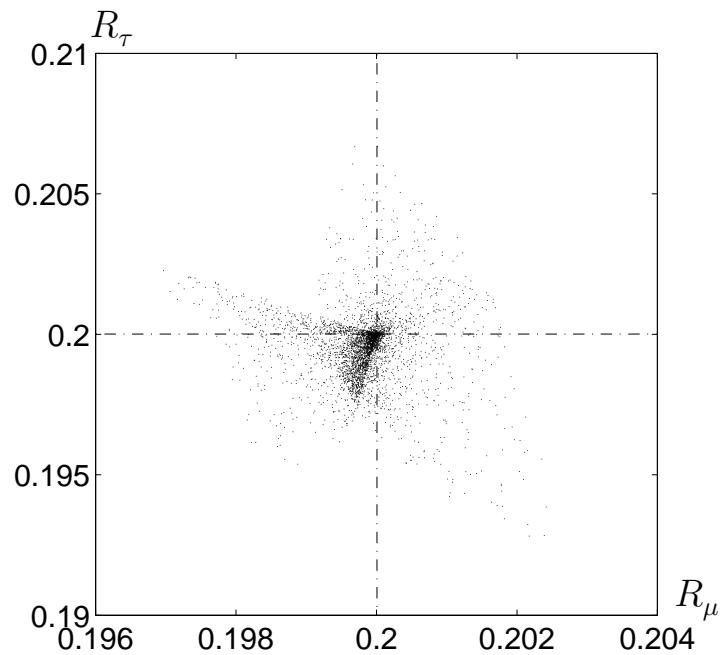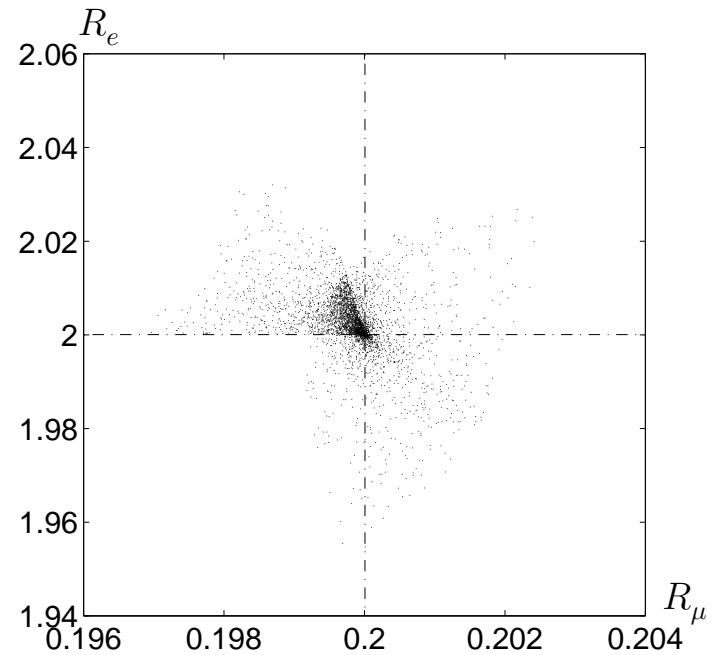
Figure 2: Allowed regions of the flavor ratios $(R_e,\ R_\mu)$ and $(R_\tau,\ R_\mu)$ in the neutrino decay scenario, where the density of points is generated by scanning the possible ranges of s_{ij} (for $ij = 14, 24, 34$) according to a flat random number distribution (i.e., $s_{ij} \in [0, 0.1]$ and $s_{14}s_{24} < 7.0 \times 10^{-5}$ based on current experimental constraints on the non-unitarity of V).